A Modal Analysis on the Irradiation Instability

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Abstract

The irradiation instability is a disk instability caused by the radiation pressure cast by a central source onto an optically thick disk. The criterion for this instability depends on a sharp transition from an optically thin inner disk to an optically thick outer disk. The quickly diminishing radiation pressure in this transition region creates a radially compressing effect, which is in many ways similar to the gravitational force. In this modal analysis we demonstrate that a disk marginally stable to irradiation can develop global modes, with growth rates being of order the dynamical timescale of the disk. The non-linear evolution of the our model shows the formation of vortices near the transition region and spiral structures propagating into the optically thick region. Consequently the scale-height of our disk's inner edge becomes time-variable and can likely be observed as a variation in its infrared flux.

The Criterion for Instability

The condition for a disk to become unstable to irradiation can be derived from a linear analysis using the WKB approximation[A]. The condition for an axisymmetric perturbation to become unstable can be written as:

$$\frac{\Omega_K^2}{\kappa^2}\beta_0 e^{-\tau}\frac{d\tau}{d\ln(r)}>1,$$

where Ω_K is the Keplerian orbital frequency, κ is the epicyclic frequency, τ is the optical depth and β_0 is the ratio of radiation pressure to gravity exerted on a mass element. It is important to note, however, that κ is not Keplerian in general as the rotation curve is modified by radiation and gas pressure. Taking into account the modification due to radiation pressure the criterion becomes:

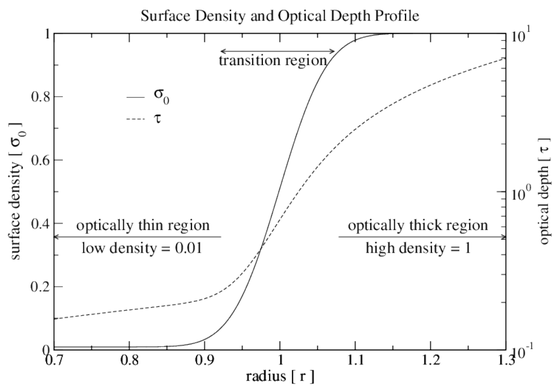
$$\frac{1-\beta_0 e^{-\tau}}{\beta_0 e^{-\tau}}-\frac{d\ln(\beta_0)}{d\ln(r)}<0.$$

This implies if β_0 is constant and less than unity this criterion is never satisfied. However we show in this study that even under this scenario global modes can still develop, similar to marginally stable self-gravitating disks. This work is currently in preparation[B].

The Setup

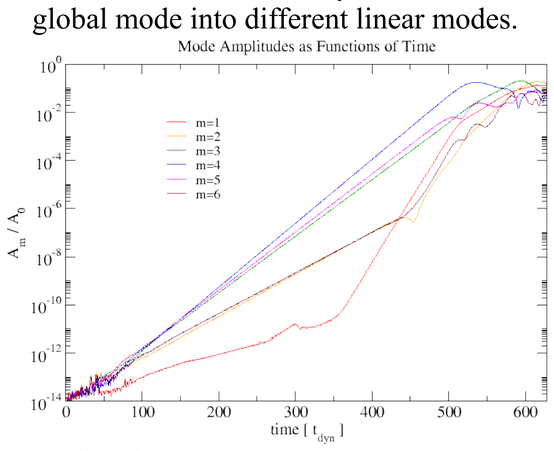
We set up a disk model to test the existence of the global modes and compute their growth rates. For simplification this disk has a uniform sound speed c_s, opacity, and β_0. This is justified by the fact that our region of interest, where the disk transitions from optically thin to think, is very narrow, with its width of order the scale height of the disk. The figure below illustrates the surface density σ_0 and τ profile.

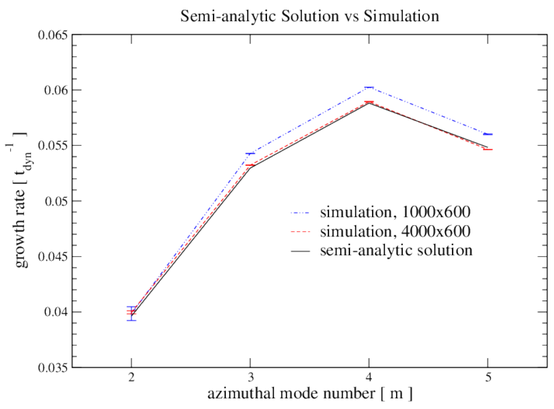
We choose a set of standard parameters dictating the disk's properties. We have c_s=0.05, β_0=0.1, and τ is normalized to 2/3 at r=1. The units are such that *GM*=1 where *M* is the mass of the host star. The transition region is created using the error function, and we define the 1-sigma width of the error function to be $2\Delta r$, where we choose Δr to be 0.05. The equation of state is isothermal.



Modal Growth

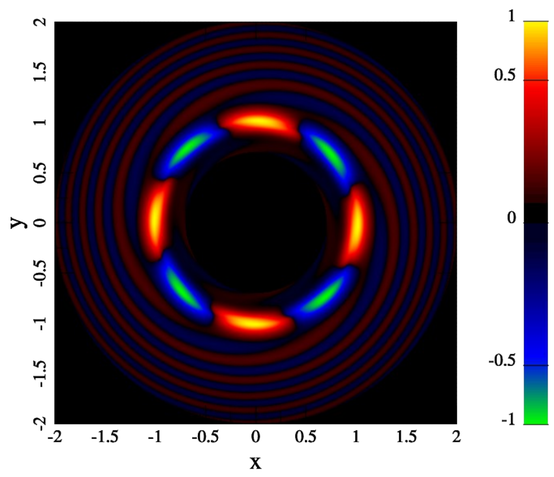
For c_s=0.05, β_0=0.1, and Δr=0.05 we compute the growth rate of the asymmetric modes by two independent methods: hydrodynamical simulation and semi-analytical solution obtained through solving the linearized equations. The azimuthal number 'm' is used to decompose the global mode into different linear modes.





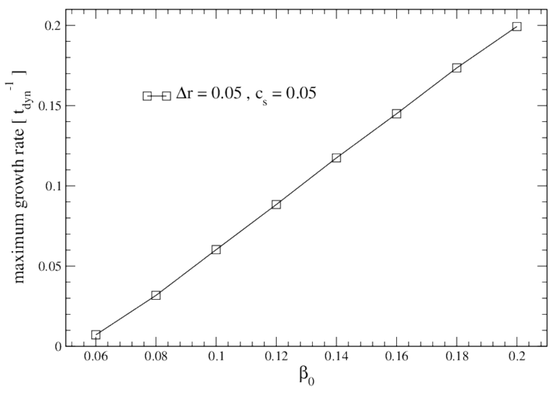
On the upper left plot is the result from a numerical simulation. There we measured the amplitude for different m[th] modes as a function of time, and observe an exponential trend with a growth rate of ~0.05 dynamical time t_{dyn}. On the upper right we show the measured growth rates from simulation and calculated growth rates from our semi-analytic solution. With higher resolution (4000 in θ and 600 in r) the agreement is almost perfect, but even with a lower resolution the agreement is still within 5%.

Top right is a normalized plot of a m=4 mode. The color represents the normalized amplitude in a square-root scale. Note that while the mode is strongest in the transition region, its tail penetrates deep into the optically thick disk. It is similar to the modes caused by the Rossby wave instability (RWI)[C,D], but our disk is stable to RWI because there would be no growth if β_0=0.

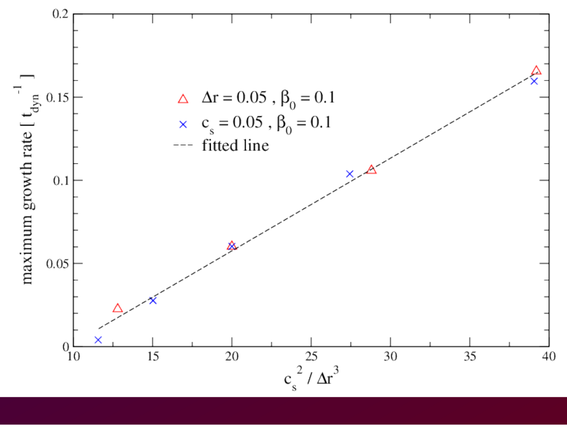


We varied three main parameters, c_s, Δr and β_0 to find out how the growth rate of the fastest growing mode scales with them. We find that not only the growth rate changes, the dominant mode also shifts to a different m. In general we observed that the dominant mode has a higher m if the growth rate is faster.

The plot in the middle right clearly shows the growth rate increases with increasing β_0 as one would expect. Below β_0=0.06 there are no measurable modal growth, confirming our hypothesis that the modal growth is driven by radiation pressure. This plot might imply that the growth rate is a linear function of β_0, but to confirm such a relation would require a more detailed study.



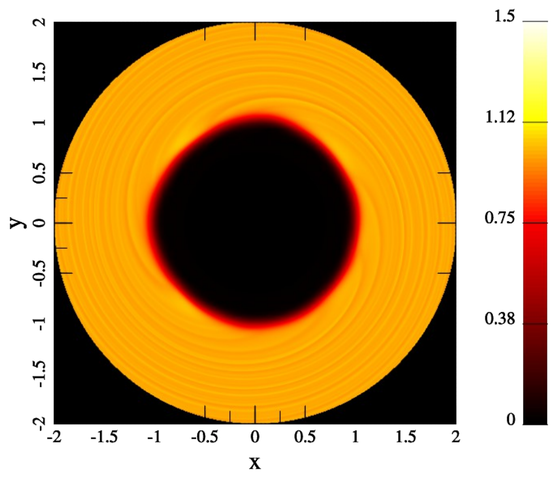
We discover a roughly linear relation between the growth rate and $c_s^2/\Delta r^3$, which is a first order approximation for the second derivative of the gas pressure. This relation is plotted bottom on the right. This suggests the growth rate is most sensitive to the width of the transition region.



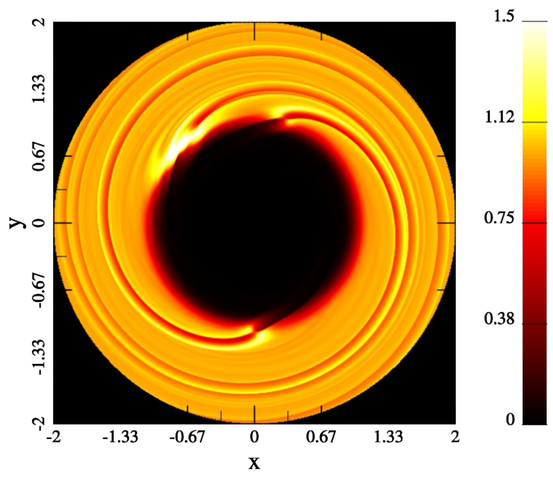
Non-linear Evolution

At the bottom we show a series of snapshots from our simulation, where the color scale represents the surface density of the disk. At t=90 orbits, or 565 t_{dyn}, the m=4 mode is visibly dominant. At 100 orbits, the modes are now fully non-linear and 3 votices are clearly visible along the transition region. These vortices travel at different frequencies, resulting in collisions and combination. At 300 orbits only 2 vortices remain. At the same time it is also noticeable that the sharp transition region has now spread, indicating the irradiation instability can put a limit to the sharpness of an inner disk edge. At 700 orbits only 1 vortex survives. It is likely that the final fate for all global modes in our models is a single vortex, regardless of the initial dominant m[th] mode. This is because the vortices generated all have the same sign and are all confined to a narrow radial region, making their combination ultimately unavoidable.

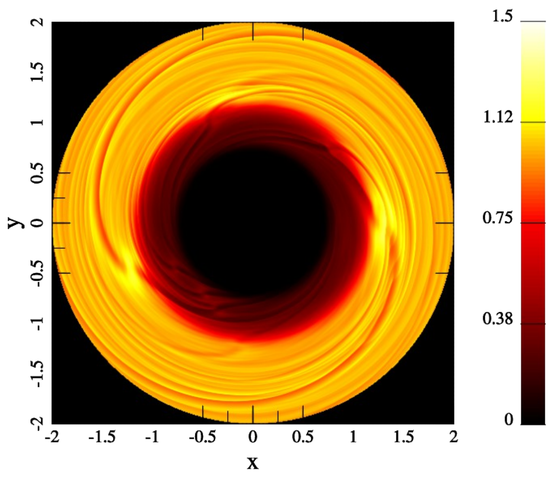
t = 090.00 orbits



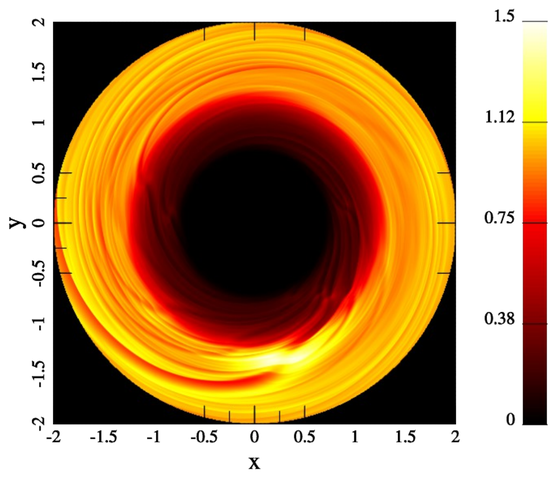
t = 100.00 orbits



t = 300.00 orbits



t = 700.00 orbits



References

A) Artymowicz, P. & Fung, J. 2013, ApJ (in prep.)

B) Fung, J. & Artymowicz, P. 2013, ApJ (in prep.)

C) Lovelace, R. V. E., Li, H., Colgate, S. A., & Nelson, A. F. 1999, ApJ, 513, 805

D) Li, H., Finn, J. M., Lovelace, R. V. E., & Colgate, S. A. 2000, ApJ, 533, 1023

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